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SOLUTION OF PROBLEM 89. (SEE PAGE 195, VOL. II.)

BY G. W. HILL.

LET x and 0 be the coordinates of the lower sphere, x' and y' those of the upper, θ and θ' the amounts of rotation, and φ the angle the line joining their centres makes with the horizon, and for brevity put $h = R + r$.

The expression for the living force is

$$T = \frac{m}{2} \left[\frac{dx^2}{dt^2} + \frac{2}{5} R^2 \frac{d\theta^2}{dt^2} \right] + \frac{m'}{2} \left[\frac{dx'^2}{dt^2} + \frac{dy'^2}{dt^2} + \frac{2}{5} r^2 \frac{d\theta'^2}{dt^2} \right],$$

and the potential is

$$\Omega = -m'gy'.$$

According to the frictional conditions, the variables x, x', y', θ and θ' satisfy the following equations,

$$\left. \begin{aligned} R\theta - x &= 0, \\ r\theta' + x + h \tan^{-1} \frac{y'}{x' - x} &= 0, \\ \sqrt{[(x' - x)^2 + y'^2]} - h &= 0. \end{aligned} \right\} \dots \dots \dots (1)$$

With Lagrange's method of multipliers, if we denote these equations respectively by $L = 0, M = 0, N = 0$, and the multipliers of their differentials by λ, μ, ν , and take ξ to represent any one of the 5 variables $x, x', y', \theta, \theta'$, the general differential equation of the problem is

$$\frac{d}{dt} \cdot \frac{dT}{d\xi} - \frac{dT}{d\xi} = \frac{d\Omega}{d\xi} + \lambda \frac{dL}{d\xi} + \mu \frac{dM}{d\xi} + \nu \frac{dN}{d\xi}.$$

Applying this in succession to each of the 5 variables and writing for simplicity φ for $\tan^{-1} \frac{y'}{x' - x}$, we get

$$\left. \begin{aligned} m \frac{d^2 x}{dt^2} &= -\lambda + \mu[1 + \sin \varphi] - \nu \cos \varphi, \\ m \frac{d^2 x'}{dt^2} &= -\mu \sin \varphi + \nu \cos \varphi, \\ m' \frac{d^2 y'}{dt^2} &= -m'g + \mu \cos \varphi + \nu \sin \varphi, \\ \frac{2}{5} m R^2 \frac{d^2 \theta}{dt^2} &= \lambda R, \\ \frac{2}{5} m' r^2 \frac{d^2 \theta'}{dt^2} &= \mu r. \end{aligned} \right\} \dots \dots \dots (2)$$

Adding the first and second of (2)

$$\frac{d^2(mx + m'x')}{dt^2} = \mu - \lambda.$$

The two first of (1) and the two last of (2) give

$$\lambda = \frac{2}{5} m R \frac{d^2 \theta}{dt^2} = \frac{2}{5} m \frac{d^2 x}{dt^2},$$

$$\mu = \frac{2}{5} m r \frac{d^2 \theta'}{dt^2} = -\frac{2}{5} m' \left[\frac{d^2 x}{dt^2} + h \frac{d^2 \varphi}{dt^2} \right].$$

Substituting these values for λ and μ in the last equation,

$$\frac{d^2(m x + m' x')}{dt^2} = -\frac{2}{5}(m + m') \frac{d^2 x}{dt^2} - \frac{2}{5} m' h \frac{d^2 \varphi}{dt^2}. \dots\dots (3)$$

Integrating once and eliminating x'

$$\frac{7}{5}(m + m') \frac{dx}{dt} + m' h \left(\frac{2}{5} - \sin \varphi \right) \frac{d\varphi}{dt} = 0,$$

where the constant is zero because the spheres are supposed to set out together from a state of rest. As $d\varphi \div dt$, in general, is negative, (φ can always be supposed in the first quadrant), it is evident from this equation, that if $\sin \varphi > \frac{2}{5}$, the lower sphere will move horizontally towards the side on which the upper sphere is; but if $\sin \varphi < \frac{2}{5}$, in the opposite direction.

Integrating (3) twice

$$(7m + 2m')x + 5m'x' + 2m'h\varphi = \text{a constant.}$$

Eliminating x and φ from this by substituting their values in terms of x' and y' , we get as the equation of the path of the centre of the upper sphere

$$7(m + m') \left[x' - \sqrt{h^2 - y'^2} \right] + m' \left[2h \sin^{-1} \frac{y'}{h} + 5 \sqrt{h^2 - y'^2} \right] = \text{a constant.}$$

As ν denotes the pressure of the upper on lower sphere, the spheres will separate when $\nu = 0$. Now if we eliminate μ between the second and third of (2), we see that $\nu = 0$ is equivalent to

$$\frac{d^2 x'}{dt^2} \cos \varphi + \left(\frac{d^2 y'}{dt^2} + g \right) \sin \varphi = 0.$$

And if we eliminate x' and y' from this by means of their values in terms of x and φ we get

$$\frac{d^2 x}{dt^2} \cos \varphi + g \sin \varphi - h \frac{d\varphi^2}{dt^2} = 0.$$

By eliminating second derivatives this becomes

$$49m \left[\frac{h}{g} \frac{d\varphi^2}{dt^2} - \sin \varphi \right] + 10m'(1 + \sin \varphi)^2 \left[\frac{h}{g} \frac{d\varphi^2}{dt^2} - 1 \right] = 0,$$

which, by substituting the value of $d\varphi^2 \div dt^2$, becomes

$$70(m + m') [49m + 10m' + 20m' \sin \varphi + 10m' \sin^2 \varphi] [\sin \beta - \sin \varphi] \\ - [10m' + (49m + 20m') \sin \varphi + 10m' \sin^2 \varphi] [49m + 45m' + 20m' \sin \varphi \\ - 25m' \sin^2 \varphi] = 0.$$